St George Girls' High School

Trial Higher School Certificate Examination

2002

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- All questions may be attempted.
- Begin each question on a new page
- All necessary working must be shown.
- Marks may be deducted for careless or poorly presented work.
- Board-approved calculators may be used.
- A list of standard integrals is included at the end of this paper.
- The mark allocated for each question is listed at the side of the question.

Students are advised that this is a Trial Examination only and does not necessarily reflect the content or format of the Higher School Certificate Examination.

Question 1 – (12 marks) – Start a new page **Marks**

a) Solve for x

$$\frac{5}{x+3} \leq 1$$ 2

b) Find the coordinates of the point that divides the interval AB with $A(-4,8)$ and $B(6, 3)$ in the ratio 3:2. 2

c) Find the exact value of $\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}}$ 2

d) Find the remainder when the polynomial $P(x) = 2x^3 - 3x$ is divided by $x + 2$. 2

e) Find $\frac{d}{dx}\left(\frac{\ln x}{x}\right)$ and hence evaluate $\int_1^2 \frac{\ln x}{x^2}\, dx$ 4

Question 2 – **(12 marks)** – **Start a new page** **Marks**

a) Show that the circle $x^2 + y^2 + 6x - 10y + 25 = 0$ touches the y-axis and give the coordinates of the point of contact. 2

b) A particle is moving in simple harmonic motion. Its displacement x at any time t is given by $x = 3\cos(2t + 5)$. 5

(i) Find the period of the motion.

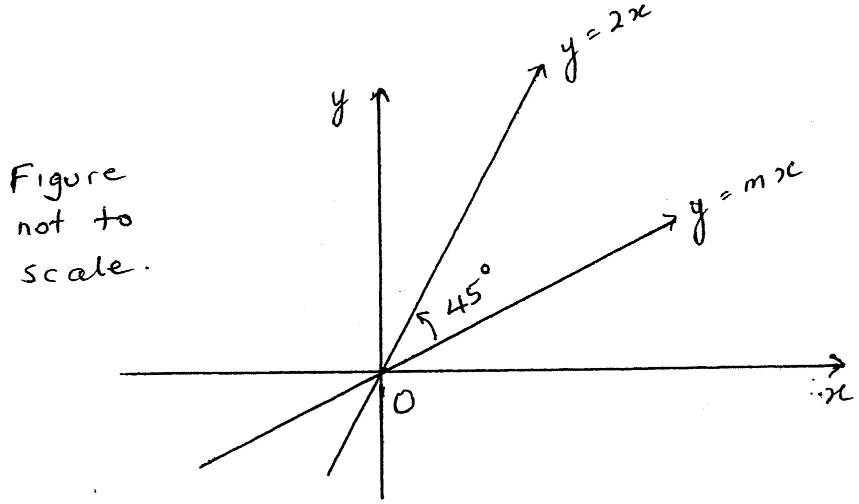
(ii) Find the maximum acceleration of the particle.

(iii) Find the speed of the particle when $x = 2$.

c) (i) Write down the expansion for $\tan(A - B)$ and use this to deduce that the acute angle θ between two lines is given by:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$ 5

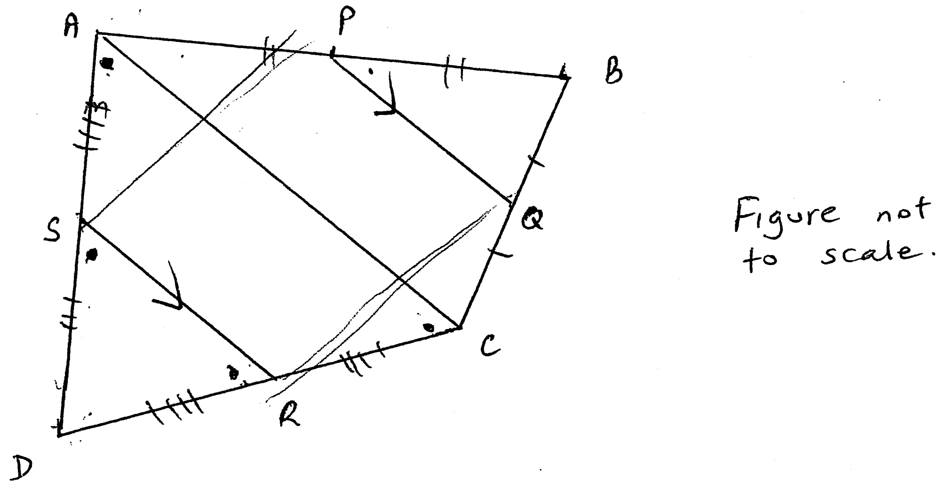
(ii)



The angle between the lines $y = 2x$ and $y = mx$ is 45° as shown in the diagram. Find the exact value of m.

Question 3 – (12 marks) – Start a new page **Marks**

a)



The sides of a quadrilateral *ABCD* have midpoints *P*, *Q*, *R* and *S* as shown.

(i) Show ΔDSR is similar to ΔDAC.

(ii) Show $RS \| QP$.

(iii) Show *PQRS* is a parallelogram.

b) Consider the function $f(x) = 2\tan^{-1} x$. 3

(i) Find the exact value of $f(\sqrt{3})$.

(ii) Find the equation of the tangent to the curve at the point where $x = \sqrt{3}$.

c) A pool holds a volume of water given by $V = 3x + 2x^2$, where x is the depth of the water. If the pool is filled with water at the rate of 0.9m^3 per hour at what rate will the level of water be increasing when the depth is 1.2m. 4

Question 4 – (12 marks) – Start a new page **Marks**

a) (i) Express $\sqrt{3}\cos x - \sin x$ in the form $R\cos(x+\alpha)$ where $R>0$ and $0 \le \alpha \le 2\pi$. 4

(ii) Hence find the general solution of $\sqrt{3}\cos x - \sin x = 1$.

b) The function $f(x) = x^2 - \ln(x+1)$ has one root between 0.5 and 1. 4

(i) Show that the root lies between 0.7 and 0.8.

(ii) Hence using halving-the-interval method find the value of the root correct to one decimal place.

c) Paula walks along a straight road. At one point she notices a tower on a bearing of 055° with an angle of elevation of 23°. After walking 240m the tower is on a bearing of 345° with an angle of elevation of 27°. 4

(i) Draw a diagram to represent the above information.

(ii) Show that the height (h) of the tower is given by

$$h^2 = \frac{240^2}{\cot^2 23° + \cot^2 27° - 2\cot 23° \cot 27° \cos 70°}$$

(iii) Calculate the height of the tower to the nearest metre.

Question 5 – (12 marks) – Start a new page

Marks

a) Consider the equation $x^3 + 2x^2 - 19x - 20 = 0$. One of the roots of this equation is equal to the sum of the other two roots.

Find the values of the three roots. 4

b) Two points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on $x^2 = 4ay$.
The normals at P and Q intersect at M 4

(i) Show the equation of the normal at P is $x + py = 2ap + ap^3$.

(ii) Show that the coordinates of M are $(-apq^2 - ap^2q,\ 2a + a(p^2 + q^2 + pq))$.

c) Yasmin invests \$$P$ at 6% p.a. compounded annually. She plans to withdraw \$4000 at the end of each year for the next 4 years to cover her son's university fees. 4

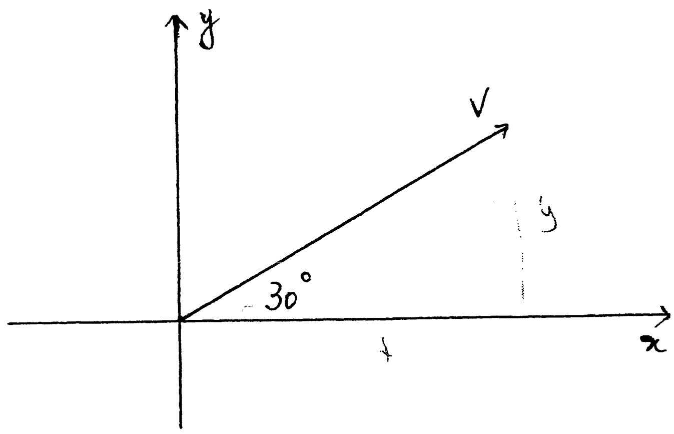
(i) Write down the amount $\$A_1$ remaining in the account following the first withdrawal.

(ii) Find an expression for the amount $\$A_2$ remaining in the account after the second withdrawal.

(iii) Calculate the amount Yasmin needs to invest if the account balance is to be \$0 after 4 years.

Question 6 – (12 marks) – Start a new page **Marks**

a) 8



A particle is projected at an angle of 30° with a velocity of v metres per second. The equations of motion of the particle are:

$$\ddot{x} = 0 \text{ and } \ddot{y} = -g$$

(i) Using calculus, derive the expression for the position of the particle at time t. Hence, show the path of the particle is given by:

$$y = \frac{x}{\sqrt{3}} - \frac{2g}{3V^2}x^2$$

A golfer hitting off from the 4th tee, strikes a ball with initial speed $v\ ms^{-1}$ and an angle of protection of 30°. The ball just clears a 3m bush which is 120m from the golfer.

(ii) Show that the initial speed of the ball is approximately 37.7 ms^{-1} (take $g = 9.8\ ms^{-2}$).

(iii) What is the horizontal distance from the bush to the point where the ball lands?

b) Prove by induction that $3^{3n} + 2^{n+2}$ is divisible by 5 for all integers $n \geq 1$. 4

Question 7 – (12 marks) – Start a new page **Marks**

a) (i) Sketch the function $f(x) = \cos^{-1} x$ 2

(ii) Find the exact value of $f\left(\frac{1}{2}\right)$. 1

(iii) Find the exact area bounded by the curve $y = f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{1}{2}$. 3

(iv) Find the volume formed if the curve $f(x) = \cos^{-1} x$ is rotated about the x-axis between $x = 0$ and $x = \frac{1}{2}$, using Simpson's Rule with 3 function values. Give your answer correct to 2 decimal places. 2

b) The acceleration a metres per second of a particle P moving in a straight line is given by $a = 1 - 9x^2$ where x metres is the displacement of the particle to the right of the origin. Initially the particle is at the origin moving with a velocity of 4 ms^{-1}. 4

(i) Show that the velocity v ms^{-1} of the particle is given by $v^2 = 16 + 2x - 6x^3$.

(ii) Will the particle ever return to the origin? Justify your answer.

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Q1

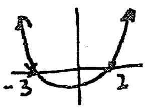
a) $\dfrac{5}{x+3} \le 1$ $\qquad x \ne -3$

$5(x+3) < (x+3)^2$

$(x+3)^2 - 5(x+3) \ge 0$

$(x+3)(x+3-5) \ge 0$

$(x+3)(x-2) \ge 0$



$x < -3 \quad x \ge 2$ $\qquad$ (2)

b) $A(-4,8) \quad B(6,3) \quad m:n = 3:2$

$x = \dfrac{mx_2 + nx_1}{m+n} \qquad y = \dfrac{my_2 + ny_1}{m+n}$

$= \dfrac{3\times 6 + 2\times -4}{5} \qquad = \dfrac{3\times 3 + 2\times 8}{5}$

$= 2 \qquad = 5$

$(2, 5)$ $\qquad$ (2)

c) $\displaystyle\int_0^{\sqrt{2}} \frac{dx}{\sqrt{4-x^2}} = \left[\sin^{-1}\frac{x}{2}\right]_0^{\sqrt{2}}$

$= \sin^{-1}\dfrac{\sqrt{2}}{2} - \sin^{-1}0$

$= \dfrac{\pi}{4}$

(2)

d) $P(x) = 2x^3 - 3x$

$P(-2) = 2(-2)^3 - 3(-2)$

$= -16 + 6$

$= -10$

e) $\dfrac{d}{dx}\left(\dfrac{\ln x}{x}\right) = \dfrac{x\frac{1}{x} - \ln x \cdot 1}{x^2}$

$= \dfrac{1 - \ln x}{x^2}$

$\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx$

$\displaystyle = -\int_1^2 \frac{-\ln x + 1 - 1}{x^2}$

$\displaystyle = -\int_1^2 \frac{1 - \ln x}{x^2} - x^{-2}\,dx$

$= -\left[\dfrac{\ln x}{x} + x^{-1}\right]_1^2$

$= -\left(\dfrac{\ln 2}{2} + \dfrac{1}{2}\right) + (0+1)$

$= -\dfrac{\ln 2}{2} + \dfrac{1}{2}$

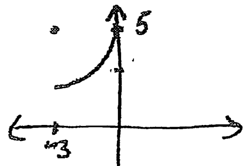
$= \dfrac{1 - \ln 2}{2}$ $\qquad$ (4)

Q2 a) $x^2+y^2+6x-10y+25=0$

$x^2+6x+9+y^2-10y+25 = -25+34$

$(x+3)^2+(y-5)^2 = 9$

circle centre $(-3,5)$ $r=3$ (2)



$(0,5)$ is the pt of contact 3 units from $(-3,5)$

b) i) $x = 3\cos(2t+5)$ $\quad T = \frac{2\pi}{n}$

$T = \frac{2\pi}{2}$ $\quad n = 2$

$= \pi$

ii) $\dot{x} = -6\sin(2t+5)$

$\ddot{x} = -12\cos(2t+5)$

max when $\cos(2t+5) = -1$

$\ddot{x} = 12$

iii) $x = 2$

$\ddot{x} = -n^2x$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -4x$

$\frac{1}{2}v^2 = -2x^2 + \frac{1}{2}C$

$v^2 = -4x^2 + C$

$C = 36$ $\quad v=0, x=3$

$v^2 = -4x^2 + 36$

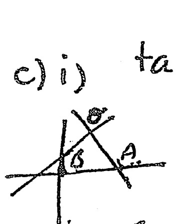
$v^2 = -16+36$ $\quad x=2$

$= 20$

$v = \sqrt{20}$

speed $= 2\sqrt{5}$ m/s.

c) i) $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$



$= \frac{m_1 - m_2}{1+m_1m_2}$

$A = \theta + B$

$\theta = A - B$

c)

ii) $\tan 45° = \left|\frac{2-m}{1+2m}\right|$ for acute angle

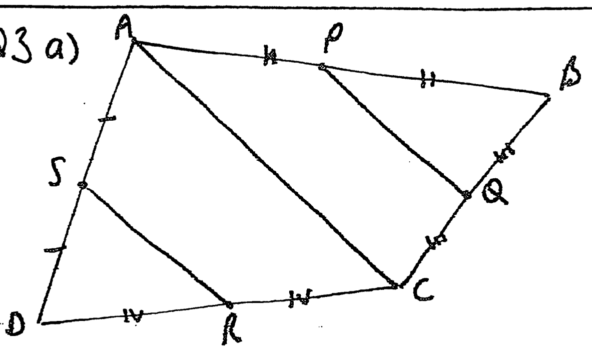
$1 = \left|\frac{2-m}{1+2m}\right|$

$1+2m = 2-m$ $\quad 1+2m = -2+m$

$3m = 1$ $\quad m = -3$

$m = \frac{1}{3}$

Q3 a)



i) In Δ's DSR ΔDAC

$DS : DA = 1:2$

$DR : DC = 1:2$

$\hat{D}$ is common

$\therefore \Delta DSR \parallel\!\mid \Delta DAC$ (2 pairs corr sides in the same ratio and the included angle equal.)

ii) Sim $\Delta BPQ \parallel\!\mid \Delta BAC$

$D\hat{S}R = D\hat{A}C$ (corr $\angle$'s of sim Δ's)

$\therefore SR \parallel AC$ (corr $\angle$'s equal)

$B\hat{P}Q = B\hat{A}C$ (corr $\angle$'s of sim Δ's)

$\therefore PQ \parallel AC$ (corr $\angle$'s equal)

$\therefore RS \parallel QP$ (parallel to same line)

(iii) $PQ : AC = 1:2$ (corr sides in sim Δ's)

$SR : AC = 1:2$

$\therefore PQ : SR = 1:1$

$\therefore PQ = SR$

$\therefore PQRS$ is a parallelogram (1 pair of sides equal and parallel)

b) i) $f(x) = 2\tan^{-1} x$

$f(\sqrt{3}) = 2\tan^{-1}\sqrt{3}$

$= 2 \cdot \frac{\pi}{3}$

$= \frac{2\pi}{3}$

ii) $f'(x) = \frac{2}{1+x^2}$ at $x = \sqrt{3}$

$= \frac{2}{4}$

$= \frac{1}{2}$

3bii) $y - \frac{2\pi}{3} = \frac{1}{2}(x - \sqrt{3})$

$2y - \frac{4\pi}{3} = x - \sqrt{3}$

$x - 2y - \sqrt{3} + \frac{4\pi}{3} = 0$

c) $V = 3x + 2x^2$ $\frac{dV}{dt} = 0.9$

$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$ $x = 1.2$

$0.9 = (3 + 4x)\frac{dx}{dt}$

$0.9 = 7.8 \frac{dx}{dt}$

$\frac{dx}{dt} = \frac{3}{26}$ m/s.

$= 0.115$ m/s

Q4 a)

$\sqrt{3}\cos x - \sin x = R\cos(x+\alpha)$

$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = \sqrt{3}$ $\quad R\sin\alpha = 1$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}}$ $\quad R = \sqrt{3+1} = 2.$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}, \quad \frac{11\pi}{6}$

$\therefore 2\cos\left(x+\frac{\pi}{6}\right) = 1$

$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$

$x+\frac{\pi}{6} = \frac{\pi}{3}$

$x+\frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$

$x = 2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}$

b) $f(x) = x^2 - \ln(x+1)$

i) $f(0.7) = 0.7^2 - \ln(1.7)$

$= -0.040628251$

$f(0.8) = 0.8^2 - \ln(1.8)$

$= 0.052213335$

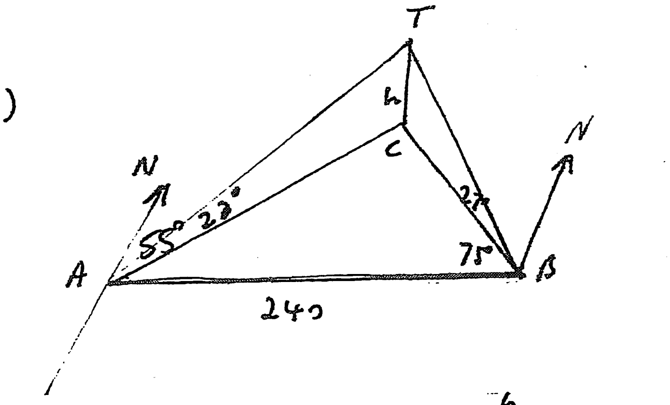
$\therefore$ root lies between 0.7 and 0.8 as there is a change in sign.

ii)

x	0.7	0.8	0.75	0.725	0.7375	0.74
y	-0.04	0.05	0.002	-0.01	-0.008	

$\therefore 0.7$ (to 1dp)

c)



In ΔATC $\quad \tan 23° = \frac{h}{AC}$

ΔBTC $\quad \tan 27° = \frac{h}{BC}$

In ΔACB

$240^2 = AC^2 + BC^2 - 2\times AC\times BC\cos(180-35-75)$

4c) $240^2 = \left(\frac{h}{\tan 23°}\right)^2 + \left(\frac{h}{\tan 27°}\right)^2 - 2\times\frac{h}{\tan 23°}\cdot\frac{h}{\tan 27}\cos 70°$

$240^2 = \frac{h^2}{\tan^2 23°} + \frac{h^2}{\tan^2 27°} - \frac{2h^2\cos 70°}{\tan 23\tan 27}$

$240^2 = h^2\cot^2 23° + h^2\cot^2 27° - 2h^2\cos\ \cot 23°\cot 27°$

$= h^2(\cot^2 23° + \cot^2 27° - 2\cos 70°\cot 23° \times \cot 27°)$

$\therefore h^2 = \frac{240}{\cot^2 23° + \cot^2 27° - 2\cos 70°\cot 23°\cot 27°}$

ii) $h = 96.0835321$

$= 96$ to nearest m.

height is 96m.

Q5

a) $x^3 + 2x^2 - 19x - 20 = 0$

$\alpha, \beta, \alpha+\beta$

$\alpha + \beta + \alpha + \beta = -\frac{b}{a}$

$2\alpha + 2\beta = -2$

$\alpha + \beta = -1$

$\alpha(\alpha+\beta) + \alpha\beta + \beta(\alpha+\beta) = \frac{c}{a}$

$\alpha^2 + \alpha\beta + \alpha\beta + \alpha\beta + \beta^2 = -19$

$\alpha^2 + 3\alpha\beta + \beta^2 = -19$

$(\alpha+\beta)^2 + \alpha\beta = -19$

$\alpha\beta = -20.$

$\alpha = -\frac{20}{\beta}$

$-\frac{20}{\beta} + \beta = -1$

$-20 + \beta^2 = -\beta$

$\beta^2 + \beta - 20 = 0$

$(\beta+5)(\beta-4) = 0$

$\beta = -5, 4$

$\alpha = 4, \beta = -5 \quad \gamma = -1$

') $P(2ap, ap^2)$ $Q(2aq, aq^2)$

$x^2 = 4ay.$

$y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$

$= \frac{x}{2a}$ at $2ap$

$y' = p$

$\therefore$ grad of normal is $-\frac{1}{p}$

$\therefore y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$

5b) ii) Similarly

eq^n at Q is

$x + qy = 2aq + aq^3$

$2ap + ap^3 - py + qy = 2aq + aq^3$

$(q-p)y = 2aq + aq^3 - 2ap - ap^3$

$= 2a(q-p) + a(q^3 - p^3)$

$y = 2a + a(q^2 + qp + p^2)$

$x + p(2a + a(q^2 + qp + p^2)) = 2ap + ap^3$

$x = \cancel{2ap} + \cancel{ap^3} - \cancel{2ap} - apq^2 - aqp^2 - \cancel{ap^3}$

$= -apq^2 - aqp^2$

$\therefore M\ (-apq^2 - ap^2q,\ 2a + a(p^2 + q^2 + pq))$

c) \$P 6%p.a annually $M = 4000$

$n = 4.$

i) $A_1 = P \times 1.06 - 4000$

ii) $A_2 = (P \times 1.06 - 4000) \times 1.06 - 4000$

$= P \times 1.06^2 - 4000 \times 1.06 - 4000$

$= P \times 1.06^2 - 4000(1.06 + 1)$

iii) $A_3 = P \times 1.06^3 - 4000(1.06^2 + 1.06) - 4000$

$= P \times 1.06^3 - 4000(1.06^2 + 1.06 + 1)$

$A_4 = P \times 1.06^4 - 4000(1.06^3 + 1.06^2 + \_ + 1)$

$A_4 = 0$

$\therefore P \times 1.06^4 - 4000(1 + 1.06 + 1.06^2 + 1.06^3) = 0$

GP $a = 1$

$r = 1.06$

$n = 4$

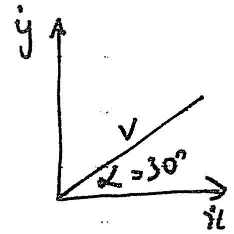
$P \times 1.06^4 = \frac{4000 \times 1(1.06^4 - 1)}{1.06 - 1}$

$P = \frac{4000 \times (1.06^4 - 1)}{1.06^4 \times 0.06}$

$= \cancel{66666.67}$ \$13860.42

$\therefore$ Principal \$~~66666.67~~.

Q6

a) 

$\dot{y} = V\sin 30$
$= \frac{V}{2}$
$\dot{x} = V\cos 30$
$= \frac{\sqrt{3}V}{2}$

i) $\ddot{x} = 0$
$\dot{x} = C_1$ $\quad \dot{x} = \frac{\sqrt{3}V}{2}$
$\dot{x} = \frac{\sqrt{3}V}{2}$

$x = \frac{\sqrt{3}Vt}{2} + C_2$ $\quad t=0$, $x=0$, $C_2=0$

$\therefore x = \frac{\sqrt{3}Vt}{2}$

$t = \frac{2x}{\sqrt{3}V}$

$\ddot{y} = -g$
$\dot{y} = -gt + C_3$ $\quad \dot{y} = \frac{V}{2}$ t=
$\dot{y} = -gt + \frac{V}{2}$ $\quad \therefore C_3 = \frac{V}{2}$

$y = \frac{-gt^2}{2} + \frac{Vt}{2} + C_4$ $\quad y=0$ t
$\therefore$

$\therefore y = \frac{-gt^2}{2} + \frac{Vt}{2}$

$\therefore y = \frac{-g\left(\frac{2x}{\sqrt{3}V}\right)^2}{2} + \frac{\cancel{V}\left(\frac{\cancel{2}x}{\sqrt{3}\cancel{V}}\right)}{\cancel{2}}$

$= -g \cdot \frac{4x^2}{6V^2} + \frac{x}{\sqrt{3}}$

$= \frac{x}{\sqrt{3}} - \frac{2g}{3V^2}x^2$

ii) $x = 120\text{m}$ $\quad y = 3$ $\quad g = 9.8$

$3 = \frac{120}{\sqrt{3}} - \frac{19.6}{3V^2}(120)^2$

$\frac{19.6}{3V^2}(120^2) = \frac{120}{\sqrt{3}} - 3$

$V^2 = \frac{19.6 \times 120^2}{3 \times \left(\frac{120}{\sqrt{3}} - 3\right)}$

$= 1419.389$

$V = 37.7$ m/s

iii) $y = 0$
$0 = \frac{x}{\sqrt{3}} - \frac{2 \times 9.8}{3 \times 1419.389}x^2$

$= 3 \times 1419.389x - 2 \times$ ... $\sqrt{3}$

$= x(3 \times 1419.389 - 2 \times 9.8 \times \sqrt{3}$

$\therefore x = 0$ $\quad x = \frac{3 \times 1419.389}{2 \times 9.8 \times \sqrt{3}}$

$x = 125.43$

$\therefore$ 5.4 metres from bust

b) Step 1 Prove true for $n=1$

LHS $= 3^3 + 2^3$
$= 27 + 8$
$= 35$
$= 5 \times 7$

$\therefore$ True for $n=1$

Step 2

Assume true for $n=k$

ie $3^{3k}+2^{k+2} = 5Q$ for all $n \geq 1$

Prove true for $n=k+1$

ie $3^{3(k+1)} + 2^{(k+1+2)} = 5K$

$$
\begin{aligned}
LHS &= 3^{3(k+1)} + 2^{(k+1+2)} \\
&= 3^{3k+3} + 2^{k+3} \\
&= 3^3(3^{3k}) + 2^{k+3} \\
&= 3^3(3^{3k} + 2^{k+2} - 2^{k+2}) + 2^{k+3} \\
&= 27(3^{3k}+2^{k+2}) - 27.2^{k+2} + 2.2^{k+2} \\
&= 27 \times 5Q - 25 \times 2^{k+2} \\
&= 5(27Q - 5.2^{k+2})
\end{aligned}
$$

$\therefore$ divisible by 5.

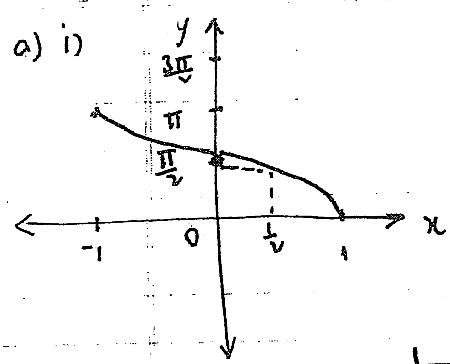
Therefore if the assertion is true for $n=k$ then it is also true for $n=k+1$

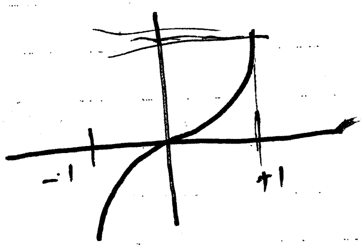
Step 3.

Since assertion is true for $n=1$ then it is true for $n=2$ and by Induction it is true for all $n \geq 1$

Q7.

a) i)





ii) $f(\tfrac{1}{2}) = \frac{\pi}{3}$

iii) Area $= \int_0^{\frac{1}{2}} \cos^{-1} x \, dx$ $\qquad y = \cos^{-1} x$, $x = \cos y$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} x \, dy + \text{rect}$

$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos y \, dy + \frac{1}{2} \times \frac{\pi}{3}$

$= [\sin y]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \frac{\pi}{6}$

$= 1 - \sin\frac{\pi}{3} + \frac{\pi}{6}$

$= 1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$

$\int_0 \sin^{-1} x \, dx$

$x = \sin y$

iv)

x	0	$\frac{1}{4}$	$\frac{1}{2}$
fx	$\frac{\pi}{2}$	1.318	$\frac{\pi}{3}$
$(fx)^2$	2.467	1.737	1.047
	$1\times$	$4\times$	$1\times$

Volume $= \frac{\frac{1}{4}}{6}\left[1 \times 2.467 + 4 \times 1.737 + 1 \times 1.047\right)$

$= 1.88558333 \times \pi$

$= 1.90 \times \pi$

v) i) $a = 1 - 9x^2$ $\qquad t = 0, x = 0, v = 4$

$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 1 - 9x^2$

$\frac{1}{2}v^2 = x - 3x^3 + C$ $\qquad v = 4 \; x = 0$

$8 = C$

$C = 8$

$\therefore \frac{1}{2}v^2 = x - 3x^3 + 8$

$v^2 = 2x - 6x^3 + 16$

$v^2 = 16 + 2x - 6x^3$

ii) $x = 0$ $\; v^2 = 16$ $\; v = \pm 4$ we know it is 4

$x = 1$ $\; v^2 = 12$ $\; v = \pm 2\sqrt{3}$

$x = 2$ $\; v^2 = -28$

$\therefore$ Between $x = 1$ and $x = 2$ $\; v = 0$

$\therefore$ velocity changes to negative

$\therefore$ particle returns to zero

as $a = 0$ at $x = \pm\frac{1}{3}$

there is a local minimum at $x = -\frac{1}{3}$

$\therefore$ x doesn't return to zero.